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MR. GLAISHER'S ENUMERATION OF PRIMES
FOR THE FIRST NINE MILLIONS.

BY PROFESSOR W. W. JOHNSON.

THE gap between Burckhardt's factor table for the first three millions and Dase's tables for the seventh, eighth and ninth millions has now been filled by the completion of Mr. James Glaisher's tables for the fourth, fifth and sixth millions, and the Report of the British Association for 1881 contains the final results of Mr. J. W. L. Glaisher's Enumeration of Primes alluded to at p. 7 Vol. V and p. 118 Vol. VII of the ANALYST.

The following tables, taken from the Report, give the number of primes in each group of 100,000.

The numbers for the complete millions are:—

	Number of Primes	Differences
First million	78,499*	
Second ,	70,433	—8,066
Third ,	67,885	—2,548
Fourth ,	66,329	—1,556
Fifth ,	65,369	— 960
Sixth ,	64,336	—1,033
Seventh ,	63,799	— 537
Eighth ,	63,158	— 641
Ninth ,	62,760	— 398

and the total number of primes in the nine millions is 602,568.

The irregular character of the distribution is strikingly exhibited by the columns of differences. Notice in the following table the occurrence of the difference + 119 between — 102 and — 90; and in the above table the sudden drop from 1,033 to 537 in the value of the difference, preceded and followed by a considerable rise in value.

Both Legendre's formula for the number of primes inferior to x , and Tchebycheff's logarithmic integral $li x$, where

$$li x = \int_0^x \frac{dx}{\log x},$$

have been shown to be hopelessly wrong. It is to be hoped that comparison will eventually be made with Riemann's formula,

$$li x - \frac{1}{2}li x^{\frac{1}{2}} - \frac{1}{3}li x^{\frac{1}{3}} - \frac{1}{5}li x^{\frac{1}{5}} + \frac{1}{6}li x^{\frac{1}{6}} - \text{etc.}, \dagger$$

which, "it seems clear, is the only one having a sound theoretical basis".

*1 and 2 are counted as primes.

†The coefficients and indices are of the form $1 + a\beta\gamma\dots$ where a, β, γ etc., are different primes and the sign is — when there is one prime, + when there are two, — when three etc.

NUMBER OF PRIMES IN EACH GROUP OF 100,000
FROM 0 TO 9,000,000.

	No. of Primes	Differ'e			No. of Primes	Differ'e
0 —	100 000	9 593	4 500 000 — 4 600 000	6 493	— 120	
100 000	200 000	8 392	— 1201	4 600 000	4 700 000	6 523 + 30
200 000	300 000	8 013	— 379	4 700 000	4 800 000	6 475 — 48
300 000	400 000	7 863	— 150	4 800 000	4 900 000	6 554 + 79
400 000	500 000	7 678	— 185	4 900 000	5 000 000	6 522 — 32
500 000	600 000	7 560	— 118	5 000 000	5 100 000	6 458 — 64
600 000	700 000	7 445	— 115	5 100 000	5 200 000	6 436 — 22
700 000	800 000	7 408	— 37	5 200 000	5 300 000	6 493 + 57
800 000	900 000	7 323	— 85	5 300 000	5 400 000	6 462 — 31
900 000	1 000 000	7 224	— 99	5 400 000	5 500 000	6 438 — 24
1 000 000	1 100 000	7 216	— 8	5 500 000	5 600 000	6 402 — 36
1 100 000	1 200 000	7 225	— 9	5 600 000	5 700 000	6 404 + 2
1 200 000	1 300 000	7 081	— 144	5 700 000	5 800 000	6 387 — 17
1 300 000	1 400 000	7 103	— 22	5 800 000	5 900 000	6 436 + 49
1 400 000	1 500 000	7 028	— 75	5 900 000	6 000 000	6 420 — 16
1 500 000	1 600 000	6 973	— 55	6 000 000	6 100 000	6 397 — 23
1 600 000	1 700 000	7 015	— 42	6 100 000	6 200 000	6 402 + 5
1 700 000	1 800 000	6 932	— 83	6 200 000	6 300 000	6 425 + 23
1 800 000	1 900 000	6 957	— 25	6 300 000	6 400 000	6 337 — 88
1 900 000	2 000 000	6 903	— 54	6 400 000	6 500 000	6 347 + 10
2 000 000	2 100 000	6 874	— 29	6 500 000	6 600 000	6 402 + 55
2 100 000	2 200 000	6 857	— 17	6 600 000	6 700 000	6 338 — 64
2 200 000	2 300 000	6 849	— 8	6 700 000	6 800 000	6 375 + 37
2 300 000	2 400 000	6 791	— 58	6 800 000	6 900 000	6 411 + 36
2 400 000	2 500 000	6 770	— 21	6 900 000	7 000 000	6 365 — 46
2 500 000	2 600 000	6 809	— 39	7 000 000	7 100 000	6 369 + 4
2 600 000	2 700 000	6 765	— 44	7 100 000	7 200 000	6 306 — 63
2 700 000	2 800 000	6 716	— 49	7 200 000	7 300 000	6 348 + 42
2 800 000	2 900 000	6 746	— 30	7 300 000	7 400 000	6 299 — 49
2 900 000	3 000 000	6 708	— 38	7 400 000	7 500 000	6 301 + 2
3 000 000	3 100 000	6 676	— 32	7 500 000	7 600 000	6 305 + 4
3 100 000	3 200 000	6 717	— 41	7 600 000	7 700 000	6 347 + 42
3 200 000	3 300 000	6 691	— 26	7 700 000	7 800 000	6 245 — 102
3 300 000	3 400 000	6 639	— 52	7 800 000	7 900 000	6 364 + 119
3 400 000	3 500 000	6 611	— 28	7 900 000	8 000 000	6 274 — 90
3 500 000	3 600 000	6 575	— 36	8 000 000	8 100 000	6 250 — 24
3 600 000	3 700 000	6 671	— 96	8 100 000	8 200 000	6 301 + 51
3 700 000	3 800 000	6 590	— 81	8 200 000	8 300 000	6 283 — 18
3 800 000	3 900 000	6 624	— 34	8 300 000	8 400 000	6 285 + 2
3 900 000	4 000 000	6 535	— 89	8 400 000	8 500 000	6 245 — 40
4 000 000	4 100 000	6 628	— 93	8 500 000	8 600 000	6 326 + 81
4 100 000	4 200 000	6 540	— 88	8 600 000	8 700 000	6 281 — 45
4 200 000	4 300 000	6 510	— 30	8 700 000	8 800 000	6 299 + 18
4 300 000	4 400 000	6 511	— 1	8 800 000	8 900 000	6 220 — 79
4 400 000	— 4 500 000	6 613	— 102	8 900 000	— 9 000 000	6 270 + 50